

Primitive uzuale 1

1. Vom calcula $\int 2x^3 + \frac{1}{\sqrt[3]{x}} + \frac{1}{3x} dx, x \in (0, \infty)$.

$$\int 2x^3 + \frac{1}{\sqrt[3]{x}} + \frac{1}{3x} dx = 2 \int x^3 dx + \int x^{-\frac{1}{3}} dx + \frac{1}{3} \int \frac{1}{x} dx = 2 \frac{x^4}{4} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{1}{3} \ln x + \mathcal{C}.$$

2. Calculăm $\int e^{\ln x} - 3^x + \frac{2}{x^3} dx, x \in (0, \infty)$.

$$\begin{aligned} \int e^{\ln x} - 3^x + \frac{2}{x^3} dx &= \int e^{\ln x} dx - \int 3^x dx + \int \frac{2}{x^3} dx = \int x dx - \int 3^x dx + 2 \int x^{-3} dx = \\ &= \frac{x^2}{2} - \frac{3^x}{\ln 3} + 2 \frac{x^{-2}}{-2} + \mathcal{C} = \frac{x^2}{2} - \frac{3^x}{\ln 3} - \frac{1}{x^2} + \mathcal{C}. \end{aligned}$$

3. Să calculăm $\int \left((3x+2)^3 - \sqrt{2x+1} \right) dx, x \in \left(-\frac{1}{2}, \infty\right)$.

Vom folosi observația:

dacă $f : J \rightarrow \mathbb{R}, J$ interval, este o funcție care admite primitive, iar F este o primitivă a sa, atunci

$$\int f(ax+b) dx = \frac{1}{a} F(ax+b) + \mathcal{C}, a \neq 0.$$

$$\int \left((3x+2)^3 - \sqrt{2x+1} \right) dx = \int (3x+2)^3 dx - \int (2x+1)^{\frac{1}{2}} dx = \frac{1}{3} \frac{(3x+2)^4}{4} - \frac{1}{2} \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} + \mathcal{C}$$