Unitatea de învățare:	Primitive

Primitive uzuale

În tabelul următor J este un interval.

1.	$f: J \to \mathbb{R}, f(x) = x^n, n \in \mathbb{N}, J \subset \mathbb{R}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + \mathscr{C}$
2.	$f: J \to \mathbb{R}, f(x) = x^a, a \in \mathbb{R} \backslash \{-1\}, J \subset (0,\infty)$	$\int x^a dx = \frac{x^{a+1}}{a+1} + \mathscr{C}$
3.	$f: J \to \mathbb{R}, f(x) = a^x, a \in (0,\infty) \backslash \{1\}, J \subset \mathbb{R}$	$\int a^x dx = \frac{a^x}{\ln a} + \mathscr{C}$
4.	$f: J \to \mathbb{R}, f(x) = \frac{1}{x}, J \subset \mathbb{R}^*$	$\int \frac{1}{x} dx = \ln \lvert x \rvert + \mathscr{C}$
5.	$f: J \to \mathbb{R}, f(x) = \sin x, J \subset \mathbb{R}$	$\int \sin x dx = -\cos x + \mathscr{C}$
6.	$f: J \to \mathbb{R}, f(x) = \cos x, J \subset \mathbb{R}$	$\int \cos x dx = \sin x + \mathscr{C}$
7.	$f: J \to \mathbb{R}, f(x) = tgx, J \subset \mathbb{R} \backslash \{(2k+1)\frac{\pi}{2} \mid k \in \mathbb{Z}\}$	$\int tgx dx = -\ln \lvert \cos x \rvert + \mathscr{C}$
8.	$f: J \to \mathbb{R}, f(x) = ctgx, J \subset \mathbb{R} \backslash \{k\pi \mid k \in \mathbb{Z}\}$	$\int ctgx dx = \ln \lvert \sin x \rvert + \mathscr{C}$
9.	$f: J \to \mathbb{R}, f(x) = \frac{1}{\cos^x}, J \subset \mathbb{R} \backslash \{(2k+1)\frac{\pi}{2} \mid k \in \mathbb{Z}\}$	$\int \frac{1}{\cos^2 x} dx = tgx + \mathscr{C}$
10.	$f: J \to \mathbb{R}, f(x) = \frac{1}{\sin^2 x}, J \subset \mathbb{R} \backslash \{k\pi \mid k \in \mathbb{Z}\}$	$\int \frac{1}{\sin x} dx = -ctgx + \mathscr{C}$
11.	$f: J \to \mathbb{R}, f(x) = \frac{1}{x^2 - a^2}, J \subset \mathbb{R} \backslash \{-a, a\}, a \neq 0$	$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \lvert \frac{x-a}{x+a} \rvert + \mathscr{C}$
12.	$f: J \to \mathbb{R}, f(x) = \frac{1}{x^2 + a^2}, J \subset \mathbb{R}, a \neq 0$	$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} arctg \frac{x}{a} + \mathscr{C}$
13.	$f: J \to \mathbb{R}, f(x) = \frac{1}{\sqrt{x^2 + a^2}}, J \subset \mathbb{R}, a \neq 0$	$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2}) + \mathscr{C}$
14.	$f: J \to \mathbb{R}, f(x) = \frac{1}{\sqrt{x^2 - a^2}}, J \subset \mathbb{R} \backslash (-a, a), a > 0$	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \lvert x + \sqrt{x^2 - a^2} \rvert + \mathscr{C}$
15.	$f: J \to \mathbb{R}, f(x) = \frac{1}{\sqrt{a^2 - x^2}}, J \subset (-a, a), a > 0$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = arcsin \frac{x}{a} + \mathscr{C}$

16.	$f: J \to \mathbb{R}, f(x) = \dfrac{x}{\sqrt{x^2+a^2}}, J \subset \mathbb{R}, a \neq 0$	$\displaystyle\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} + \mathscr{C}$
17.	$f: J \to \mathbb{R}, f(x) = \dfrac{x}{\sqrt{a^2-x^2}}, J \subset (-a, a), a > 0$	$\displaystyle\int \frac{x}{\sqrt{x^2+a^2}} dx = -\sqrt{a^2-x^2} + \mathscr{C}$

Exemple

1. $\displaystyle\int \frac{1}{4x^2+1} dx = \frac{1}{4}\int \frac{1}{x^2+\frac{1}{4}} dx = \frac{1}{4}\frac{1}{\frac{1}{2}} arctg \frac{x}{\frac{1}{2}} + \mathscr{C} = \frac{1}{2} arctg 2x + \mathscr{C};$

2. $\displaystyle\int \frac{x+1}{\sqrt{4-x^2}} dx = \int \frac{x}{\sqrt{4-x^2}} + \frac{1}{\sqrt{4-x^2}} dx = -\sqrt{4-x^2} + arcsin \frac{x}{2} + \mathscr{C};$

3. $\displaystyle\int \frac{1}{\sin 2x} dx = \int \frac{1}{2\sin x \cos x} dx = \frac{1}{2}\int \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} dx = \frac{1}{2}\left(\int \frac{\sin^2 x}{\sin x \cos x} dx + \int \frac{\cos^2 x}{\sin x \cos x} dx\right)$
$\displaystyle= \frac{1}{2}\left(\int \frac{\sin x}{\cos x} dx + \int \frac{\cos x}{\sin x} dx\right) = \frac{1}{2}\left(\int tgx dx + \int ctgx dx\right) = \frac{1}{2}\left(-\ln|\cos x| + \ln|\sin x|\right) + \mathscr{C}$

Observație

Dacă $f: J \to \mathbb{R}$, J interval, este o funcție care admite primitive, iar F este o primitivă a sa, atunci $\displaystyle\int f(ax+b)dx = \frac{1}{a}F(ax+b) + \mathscr{C}, a \neq 0.$

Exemple

1. $\displaystyle\int \cos 2x dx = \frac{1}{2}\sin x + \mathscr{C};$
2. $\displaystyle\int e^{3x} dx = \frac{1}{3}e^{3x} + \mathscr{C};$
3. $\displaystyle\int \frac{1}{4x+3} dx = \frac{1}{4}\ln|4x+3| + \mathscr{C};$
4. $\displaystyle\int \sin \frac{x}{2} dx = = 2\cos \frac{x}{2} + \mathscr{C}$